Reminders:

1) Quiz tomorrow (over pre-calculus/algebra), new practice quiz up

2) Choose office hours on Doodle calendar

Look at problems

Limits (Sections 1.4 – 1.7)

Limits revolutionized math/physics and led to the development of the derivative and integral.

They allow you to quantify notions like "infinitesimally small" or "as close as you like" or even "infinite time".

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Example: (tumor growth)

In January, a brain cancer (glioma) tumor is the size of a pin. In July, it is the size of a golf ball.

A doctor can tell you the average rate of change, but what if you need to have greater accuracy?

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Limits can help you determine the instantaneous rate of change. The following pictures for a function $y=f(x)$ will hopefully give some intuition on how limits work. Think of it as the y-values that the function "tends to".

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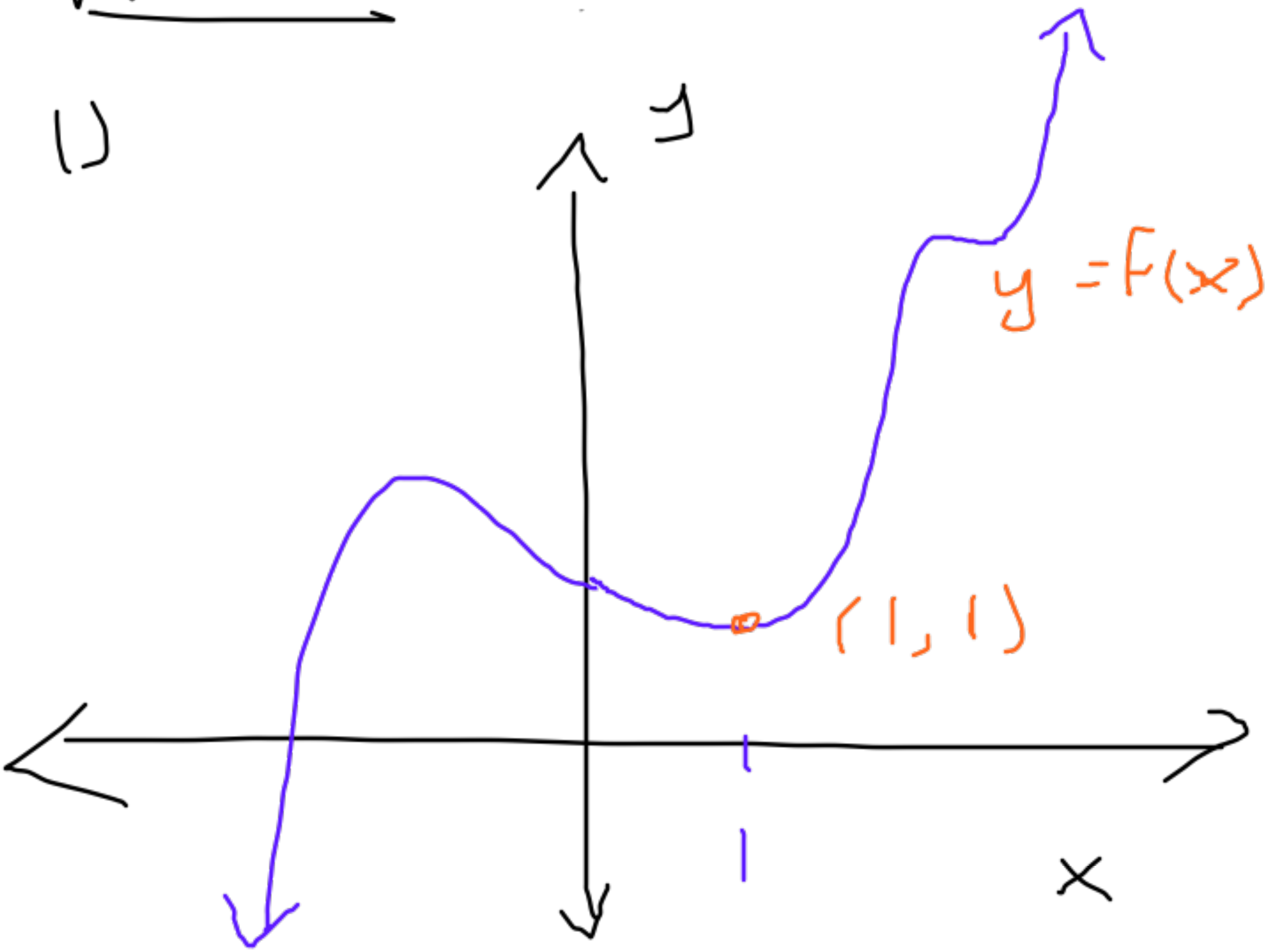
Pictures

1)



As you get closer to x = 1 from either the left or right the y-values "tend to" y = 1.

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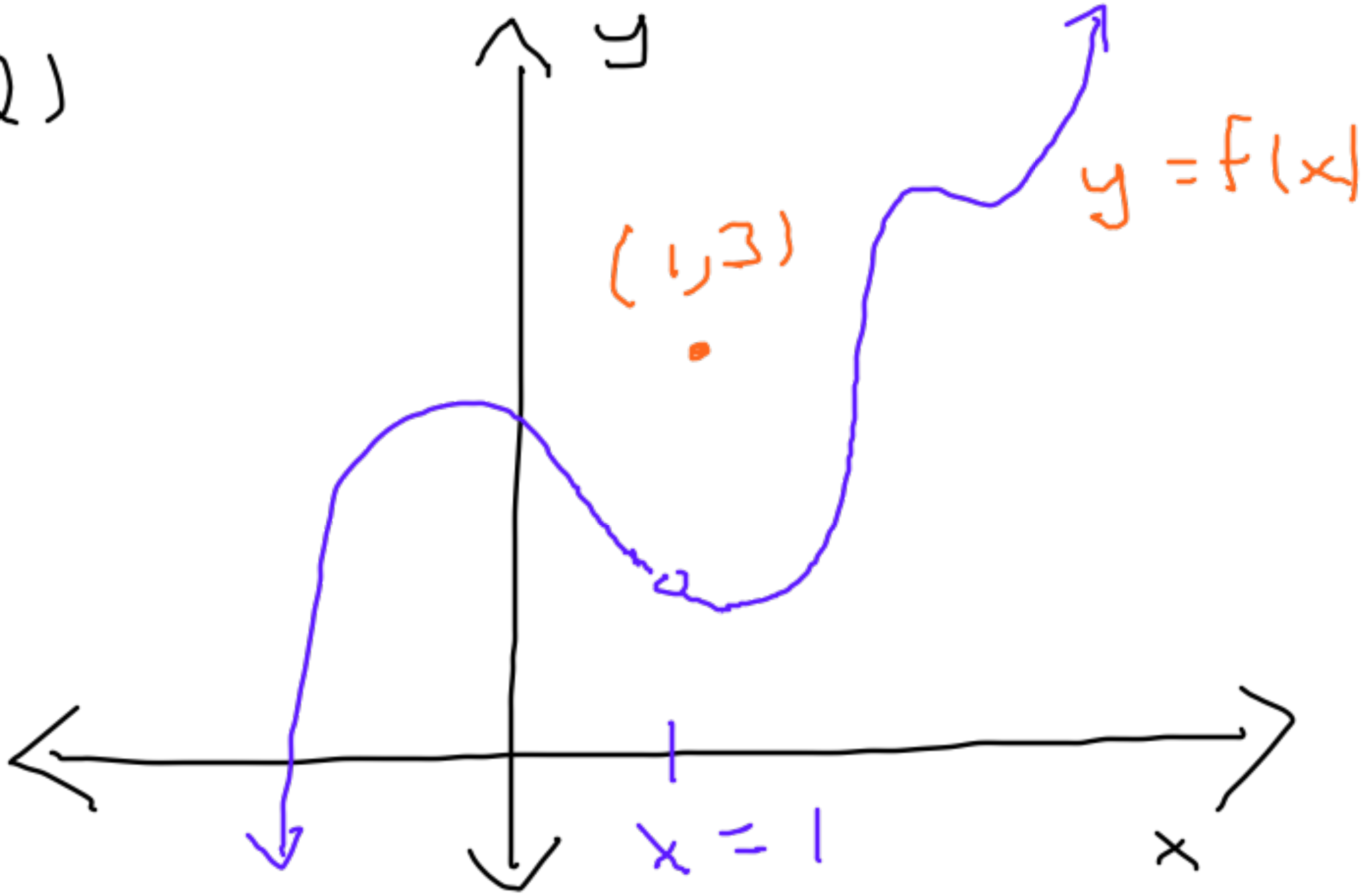
2)



Even though $f(1) = 3$, the y-values still "tend to" $y = 1$ as you get closer to $x = 1$ from the right or left.

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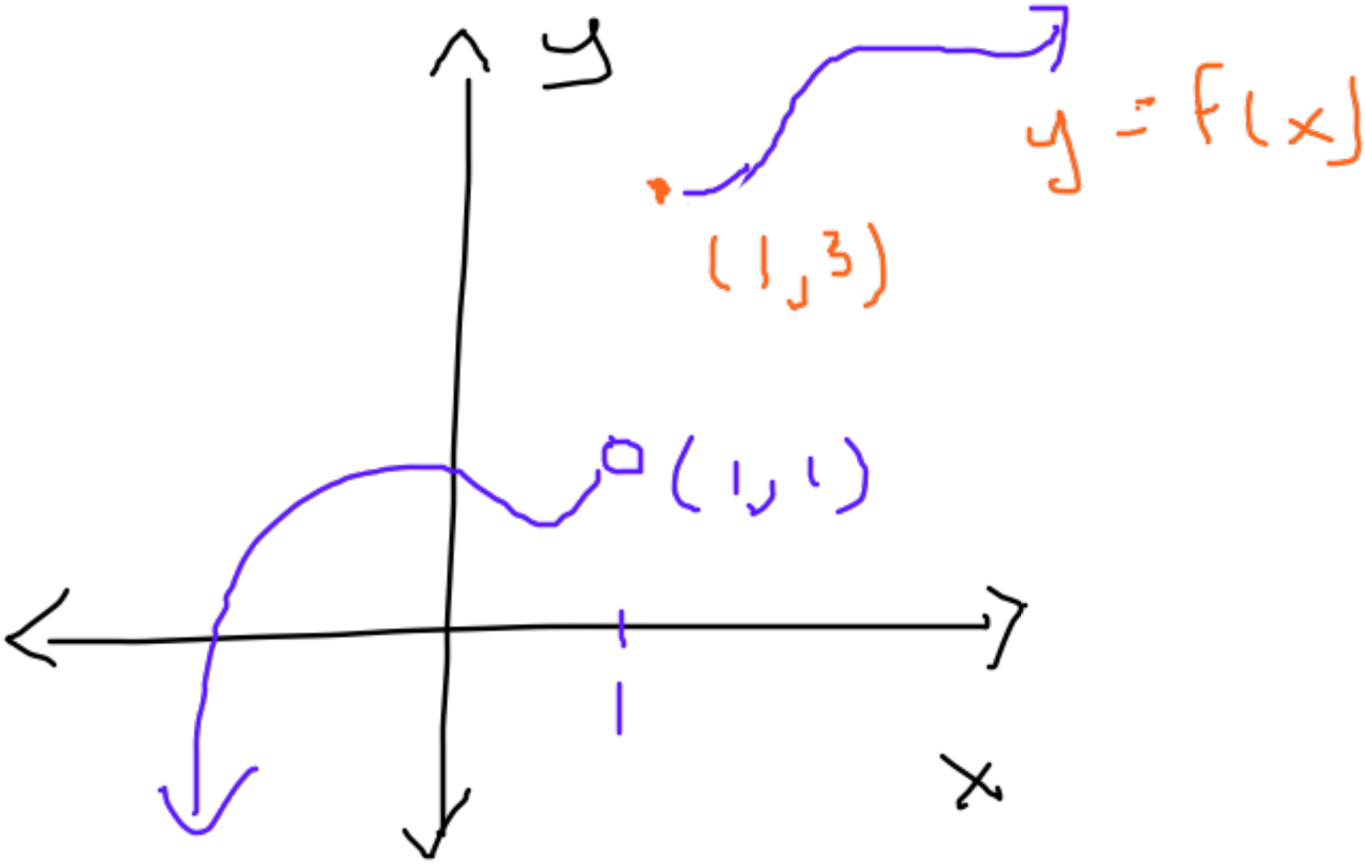
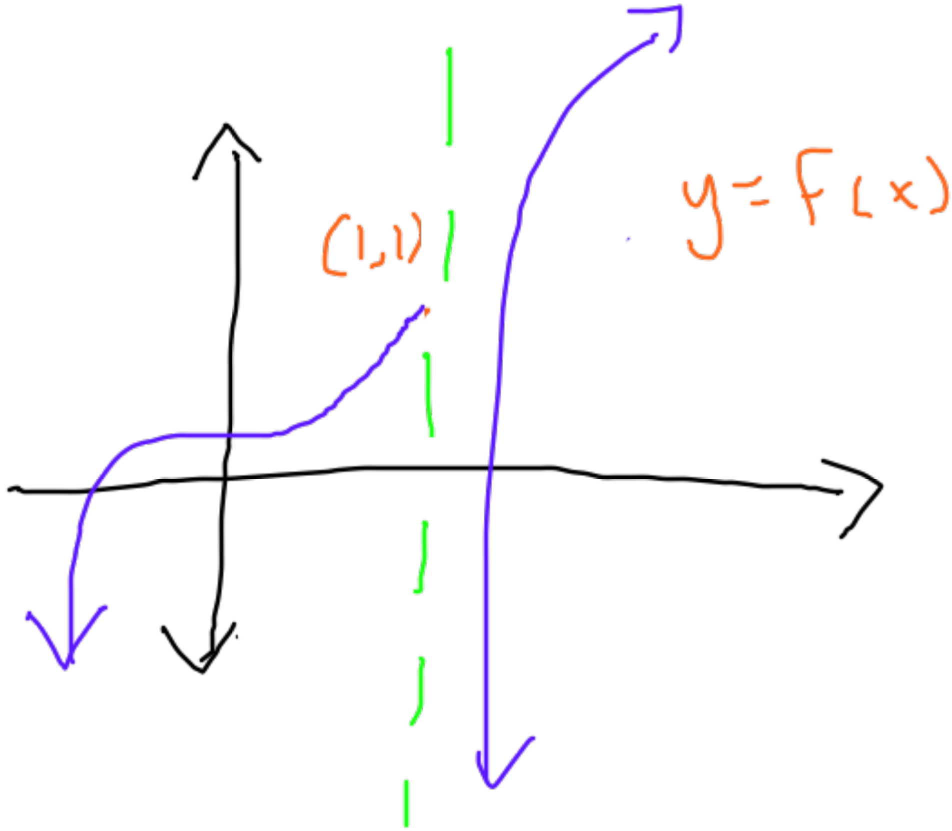
3)



Now as you get closer to $x = 1$ from the left, the y-values "tend to" $y=1$, but from the right, the y-values "tend to" $y=3$

4)



Coming towards x=1 from the left, the y-values "tend to" y=1, but from the right, they don't "tend to" any finite number.

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Precise Definitions

(and notation)

$y = f(x)$ a function.

$x = a$ some number

L a real number.

Limit from the left: The limit from the left of $f(x)$ at $x = a$ is equal to L (denoted $\lim_{x \to a^-} f(x) = L$) if for every $\varepsilon > 0$, there is a $\delta > 0$ such that $|f(x) - L| < \varepsilon$ when

$0 < a - x < \delta$.

If there is no number L, we say that the limit does not exist.

Limit from the right

The limit from the right of $f(x)$ at $x=a$ is equal to L (denoted $\lim_{x \to a^+} f(x) = L$)

if for every $\varepsilon > 0$, there is a $\delta > 0$ such that

$|f(x) - L| < \varepsilon$ when

$0 < x - a < \delta$

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Limit If

$$\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$$

then we say the limit of $f(x)$ is equal to L and write $\lim_{x \to a} f(x) = L$.

Translations:

ε = some small number

δ = whatever number sends x-values δ close to $x=a$ to y-values ε close to L.

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Picture of $\lim_{x \to a} f(x) = L$